Capacity Analysis of an Opportunistic Scheduling System in a Spectrum Sharing Environment

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***Abstract*—We analyze the capacity of an opportunistic scheduling system in a spectrum sharing environment where multiple secondary users can share a frequency spectrum with multiple primary users as long as secondary users do not cause interference power exceeding a given threshold to the primary users. We consider three different power control schemes of secondary users: fixed transmit power, adaptive transmit power, and infinite transmit power schemes. Our numerical and simulation results show that the capacity of the adaptive transmit power scheme is similar to that of the fixed transmit power scheme in the low transmit power region, while the capacity of the adaptive transmit power scheme is close to that of the infinite transmit power scheme in the high transmit power region and is saturated beyond a certain point.**

I. Introduction

Demands for wider frequency spectra have been dramatically increasing because mobile Internet traffic is continuously increasing and new wireless mobile applications emerge, while the given spectrum is a limited resource and the spectrum utilization is generally very low because it is exclusively licensed for a single purpose across wide regions. Some measurement reports clearly indicate that some portion of the allocated spectrum is never accessed or is accessed for only a fraction of time in a certain area [1] – [3]. In order to mitigate this spectrum utilization problem, the spectrum policy task force (SPTF) presented a concept of *spectrum sharing* [1]. With this spectrum sharing technique, secondary users can share a spectrum already allocated to primary users as long as the secondary users' spectrum sharing interferes the primary users' operation within an allowable interference power constraint. Generally, the quality of primary users' communications is degraded by the interference power from secondary users sharing the primary users' spectrum. Thus, in order to control the interference power from secondary users to primary users, SPTF [1] introduced a concept of *interference temperature*, which represents the maximum permissible level of interference power received at the primary users. This spectrum sharing mechanism using the interference temperature concept is one application of cognitive radio (CR) which is one of promising technologies in next generation wireless communication systems [3] – [5].

Based on this motivation, there have been several studies on the spectrum sharing [6] – [8]. Gastpar [6] investigated the channel capacity under a scenario where the received interference power at a primary user's receiver is maintained within a given constraint. Although this result is very meaningful, it did not consider channel fading, which inevitably occurs in real wireless environments and has a significant effect on channel capacity [9], [10]. Ghasemi and Sousa [7] analyzed the capacity of a spectrum sharing scheme considering the effect of channel fading and compared the capacity for Rayleigh and Nakagami fading models. They also considered multiple primary users and showed that significant capacity gains can be achieved when channels are varying due to fading. Gastpar [6] and Ghasemi and Sousa [7] assumed that both primary and secondary tranceivers have a single antenna each, and Zhang [8] investigated the channel capacity of a spectrum sharing system considering multiple antennas and showed that a significant capacity gain can be achieved by using multiple antennas. Although previous studies analyzed the fundamental channel capacity of spectrum sharing systems under various scenarios, they did not consider multiple secondary users, while it is well known that the performance gain can be achieved through opportunistic scheduling in multi-user environments by exploiting channel fluctuations [11] – [13].

In this context, we analyze the capacity of an opportunistic scheduling system in a spectrum sharing environment where there exist multiple secondary users. The rest of this paper is organized as follows. In Section II, both a system model and three different power control schemes of secondary transmitters are described. In Section III, the average achievable capacity of the three different power control schemes is mathematically analyzed. In Section IV, numerical results are shown. Finally, conclusions are drawn in Section V.

II. System Model and Transmission Power Control Strategies

Fig. 1 shows a communication system model where N_s secondary transmitters can share a spectrum which is primarily licensed to N_p primary receivers. Any data transmission of secondary transmitters should be maintained to cause interference power to primary users within a given interference temperature, Q, which represents the maximum allowable interference power level. $\alpha_{i,j}$ and β_i denote the interference channel gain from the i-th secondary transmitter to the j-th primary receiver and data channel gain from the i-th secondary transmitter to a secondary receiver, respectively. They are

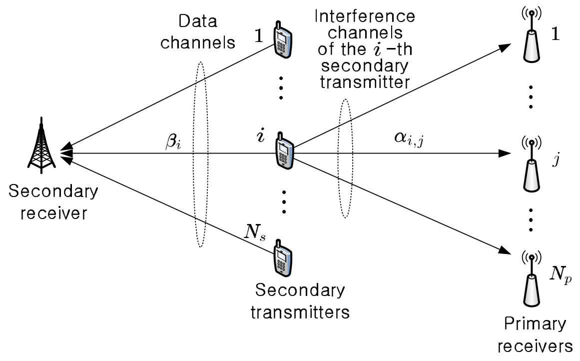

Fig. 1. System model

assumed to be exponentially distributed random variables with unit mean and independent and identically distributed (i.i.d). It is also assumed that the i-th secondary transmitter has information about $\alpha_{i,j}$, which can be obtained through measurement of sounding channels from primary receivers.

In addition, we consider three different power control schemes for secondary transmitters according to power control capability and peak power constraints: fixed transmit power, adaptive transmit power, and infinite transmit power schemes. In the fixed transmit power scheme which is suitable for secondary transmitters that require low complexity, each secondary transmitter can transmit data with its peak power, P, when its transmission does not cause interference power exceeding Q to any primary receiver, while the secondary user can not transmit data if its transmission causes interference exceeding Q to any primary receiver.

On the other hand, in the adaptive transmit power scheme, a secondary transmitter can adaptively adjust its transmit power level if its transmission causes interference power exceeding Q to any primary receiver. Thus, the adaptive transmit power scheme is more complicated, compared to the fixed transmit power scheme. Finally, we consider the infinite transmit power scheme where secondary transmitters can adaptively adjust their transmit power level and the peak transmit power of the secondary transmitters is assumed to be infinite. This scheme can yield a theoretically optimal capacity bound although it is not practical.

For these three different transmit power control schemes, each secondary transmitter computes its maximum permissible transmit power level and reports it to a secondary receiver. Then, the secondary receiver can obtain the received signal-to-noise power ratio (SNR) for each secondary transmitter and selects one secondary transmitter with the best received SNR in order to receive data.

III. Capacity Analysis

A. Fixed Transmit Power Scheme

In the fixed transmit power scheme, the transmit power of the i-th secondary transmitter is given by

$$P_i^F = \begin{cases} P , & \alpha_i \leq \frac{Q}{P} \\ 0 , & \alpha_i > \frac{Q}{P}, \end{cases} \tag{1}$$

where α_i denotes the effective interference channel gain of the i-th secondary transmitter and is defined as $\max_{1\leq j\leq N_p} \alpha_{i,j}$. The probability density function (PDF) of α_i can be obtained as [14]

$$f_{\alpha_i}(x) = N_p e^{-x}\left(1-e^{-x}\right)^{N_p-1}.$$

Then, the received SNR of the secondary transmitter can be represented as

$$\gamma_i^F = \frac{P_i^F \beta_i}{\sigma^2} = \begin{cases} P\beta_i , & \alpha_i \leq \frac{Q}{P} \\ 0 , & \alpha_i > \frac{Q}{P}, \end{cases} \tag{2}$$

where σ^2 denotes the variance of white gaussian noise and is set to be one for simplicity of mathematical analysis so that P and Q can also be considered as the transmit SNR and interference temperature-to-noise power ratio, respectively. Eqs. (1) and (2) indicate that a secondary transmitter does not transmit any data if its transmission cause interference power exceeding Q to any primary user. Otherwise, the secondary transmitter can transmit data with its peak power. Thus, if we consider a set, $\mathcal{S}_n(|\mathcal{S}_n| = n, 0 \leq n \leq N_s)$, which consists of secondary transmitters which can transmit data, the PDF and CDF of each secondary user's received SNR in the set can be obtained as

$$f_{\gamma_i^F}(x) = \frac{1}{P}e^{-\frac{x}{P}}, \quad i \in \mathcal{S}_n \tag{3}$$

$$F_{\gamma_i^F}(x) = 1-e^{-\frac{x}{P}}, \quad i \in \mathcal{S}_n. \tag{4}$$

Then, a secondary receiver selects one secondary transmitter with the best received SNR in $\mathcal{S}_n$. The received SNR at the secondary receiver from the selected secondary transmitter can be obtained as

$$\gamma_{max}^F = \max_{i\in\mathcal{S}_n} \gamma_i^F, \tag{5}$$

and its PDF is given by

$$\begin{aligned} f_{\gamma_{max}^F}(x) &= n f_{\gamma_i^F}(x)\left(F_{\gamma_i^F}(x)\right)^{n-1} \\ &= \frac{n}{P}e^{-\frac{x}{P}}\left(1-e^{-\frac{x}{P}}\right)^{n-1}. \end{aligned} \tag{6}$$

Using Eq. (6), for a given n, the achievable capacity of the fixed transmit power scheme can be derived as

$$\begin{aligned} C_n &\triangleq \mathbb{E}\left[\log_2(1+\gamma_{max}^F)\right] \\ &= \int_0^\infty \log_2(1+x) f_{\gamma_{max}^F}(x)dx \\ &= \frac{n}{P}\int_0^\infty \log_2(1+x)\, e^{-\frac{x}{P}}\left(1-e^{-\frac{x}{P}}\right)^{n-1}dx \\ &= \frac{n}{P}\int_0^\infty \log_2(1+x)\, e^{-\frac{x}{P}}\sum_{k=0}^{n-1}\binom{n-1}{k}(-1)^k e^{-\frac{kx}{P}}dx \\ &= \frac{n}{P}\sum_{k=0}^{n-1}\binom{n-1}{k}(-1)^k\int_0^\infty \log_2(1+x)\, e^{-\frac{(k+1)}{P}x}dx \\ &= n\log_2(e)\sum_{k=0}^{n-1}\binom{n-1}{k}\frac{(-1)^k}{k+1}e^{\frac{k+1}{P}}E_1\left(\frac{k+1}{P}\right). \end{aligned} \tag{7}$$

Averaging C_n in Eq. (7) over n, the overall average achievable capacity of the fixed transmit power scheme can be expressed as

$$C^F = \mathbb{E}[C_n] = \sum_{n=0}^{N_s} P_n \times C_n, \tag{8}$$

where P_n is the probability that the cardinality of $\mathcal{S}_n$ is n and can be obtained as

$$P_n = \binom{N_s}{n} \left(1 - e^{-\frac{Q}{P}}\right)^{nN_p} \left(1 - \left(1 - e^{-\frac{Q}{P}}\right)^{N_p}\right)^{N_s - n}.$$

B. Adaptive Transmit Power Scheme

In the adaptive transmit power scheme, each secondary transmitter adaptively adjusts its transmit power within its peak power constraint so that a given interference temperature constraint is satisfied. Thus, the received SNR at a secondary receiver from the i-th secondary transmitter can be described as

$$\gamma_i^A = \begin{cases} P\beta_i \ , & \alpha_i \le \frac{Q}{P} \\ \frac{Q\beta_i}{\alpha_i} \ , & \alpha_i > \frac{Q}{P}, \end{cases} \tag{9}$$

and its CDF can be expressed as

$$\begin{aligned} F_{\gamma_i^A}(x) &= \Pr\left[\alpha_i \le \frac{Q}{P}\right]\left(1 - e^{-\frac{x}{P}}\right) \\ &\quad + \Pr\left[\frac{Q\beta_i}{\alpha_i} \le x \middle| \alpha_i > \frac{Q}{P}\right], \end{aligned} \tag{10}$$

where $\Pr\left[\alpha_i \le \frac{Q}{P}\right]$ can be derived as

$$\Pr\left[\alpha_i \le \frac{Q}{P}\right] = \left(1 - e^{-\frac{Q}{P}}\right)^{N_p},$$

and $\Pr\left[\frac{Q\beta_i}{\alpha_i} \le x \middle| \alpha_i > \frac{Q}{P}\right]$ can be obtained, using the result of Appendix, as

$$\begin{aligned} &\Pr\left[\frac{Q\beta_i}{\alpha_i} \le x \middle| \alpha_i > \frac{Q}{P}\right] \\ &= N_p \sum_{j=1}^{N_p} \binom{N_p - 1}{j - 1} (-1)^{j-1} e^{-\frac{Qj}{P}} \left[\frac{1}{j} - \frac{Q}{Qj + x} e^{-\frac{x}{P}}\right]. \end{aligned}$$

Thus, Eq. (10) can be derived as

$$\begin{aligned} &F_{\gamma_i^A}(x) = \left(1 - e^{-\frac{Q}{P}}\right)^{N_p}\left(1 - e^{-\frac{x}{P}}\right) + \\ &N_p \sum_{j=1}^{N_p} \binom{N_p - 1}{j - 1} (-1)^{j-1} e^{-\frac{Qj}{P}} \left[\frac{1}{j} - \frac{Q}{Qj + x} e^{-\frac{x}{P}}\right]. \end{aligned} \tag{11}$$

It follows that the PDF of γ_i^A can be obtained as

$$\begin{aligned} &f_{\gamma_i^A}(x) = \frac{1}{P}\left(1 - e^{-\frac{Q}{P}}\right)^{N_p} e^{-\frac{x}{P}} + QN_p e^{-\frac{x}{P}} \times \\ &\sum_{j=1}^{N_p} \binom{N_p - 1}{j - 1} (-1)^{j-1} e^{-\frac{Qj}{P}} \left[\frac{P + Qj + x}{P\,(Qj + x)^2}\right]. \end{aligned} \tag{12}$$

Contrary to the fixed transmit power scheme, all secondary transmitters can always transmit data because they can adaptively reduce their transmit power satisfying a given interference temperature constraint. Thus, a secondary receiver selects one secondary transmitter with the best received SNR among N_s secondary transmitters. The received SNR at the secondary receiver from the selected secondary transmitter can be described as

$$\gamma_{max}^A = \max_{0 \le i \le N_s} \gamma_i^A. \tag{13}$$

Its PDF can be obtained as

$$f_{\gamma_{max}^A}(x) = N_s f_{\gamma_i^A}(x) \left(F_{\gamma_i^A}(x)\right)^{N_s - 1}. \tag{14}$$

Using Eq. (14), the overall average achievable capacity can be expressed and numerically calculated as

$$\begin{aligned} C^A &= \mathbb{E}\left[\log_2(1 + \gamma_{max}^A)\right] \\ &= \int_0^\infty \log_2(1 + x)\, f_{\gamma_{max}^A}(x) dx. \end{aligned} \tag{15}$$

C. Infinite Transmit Power Scheme

In the infinite transmit power scheme, secondary transmitters do not have a peak power constraint, that is, they can use infinite transmit power. Thus, the received SNR at a secondary receiver from the i-th secondary transmitter can be described as

$$\gamma_i^I = \frac{Q\beta_i}{\alpha_i}. \tag{16}$$

Its CDF and PDF can be derived as

$$\begin{aligned} F_{\gamma_i^I}(x) &= \Pr\left[\frac{Q\beta_i}{\alpha_i} \le x\right] \\ &= N_p \sum_{j=1}^{N_p} \binom{N_p - 1}{j - 1} (-1)^{j-1} \left[\frac{1}{j} - \frac{Q}{Qj + x}\right], \end{aligned} \tag{17}$$

$$f_{\gamma_i^I}(x) = QN_p \sum_{j=1}^{N_p} \binom{N_p - 1}{j - 1} \frac{(-1)^{j-1}}{(Qj + x)^2}, \tag{18}$$

where Eq. (17) is derived by replacing A with 0 in Appendix. In this scheme, all secondary transmitters can transmit data. Thus, a secondary receiver selects one secondary transmitter with the best received SNR among N_s secondary transmitters. The received SNR at the secondary receiver from the selected secondary transmitter can be described as

$$\gamma_{max}^I = \max_{0 \le i \le N_s} \gamma_i^I, \tag{19}$$

and its PDF can be obtained as

$$f_{\gamma_{max}^I}(x) = N_s f_{\gamma_i^I}(x) \left(F_{\gamma_i^I}(x)\right)^{N_s - 1}. \tag{20}$$

Using Eq. (20), the overall average capacity can be obtained and numerically calculated as

$$\begin{aligned} C^I &= \mathbb{E}\left[\log_2(1 + \gamma_{max}^I)\right] \\ &= \int_0^\infty \log_2(1 + x)\, f_{\gamma_{max}^I}(x) dx. \end{aligned} \tag{21}$$

IV. Numerical Results

Fig. 2 shows the average achievable capacity versus the peak power of secondary transmitters when $Q = 0$ or 3dB and $N_s = N_p = 10$. Monte-Carlo simulation results are also shown in order to verify our numerical analysis. When the peak transmit power of secondary transmitters is sufficiently low compared to a given interference temperature, the fixed transmit power and adaptive transmit power schemes achieve almost the same capacity because secondary transmitters can use their available peak power with high probability, while the probability that secondary transmitters can not use their peak power is negligible. On the other hand, when the peak power of secondary transmitters is high compared to a given interference temperature, the capacity of the adaptive transmit power scheme approaches that of the infinite transmit power scheme and is saturated beyond a certain point as P increases further. As P increases, the capacity of the fixed transmit power scheme is seriously degraded because the average number of secondary transmitters which can transmit data becomes small.

Fig. 3 shows the average achievable capacity versus N_s. It is shown that all capacities of the three schemes increase as N_s increases because of an increased multi-user diversity gain. It is also verified that the capacity of the adaptive transmit power scheme is similar to that of the fixed transmit power scheme for $P = -10$dB, while the capacity of the adaptive transmit power scheme is close to that of the infinite transmit power scheme for $P = 0$dB.

Fig. 4 shows the average achievable capacity versus N_p. The achievable capacities decrease as N_p increases because an increase in the number of primary receivers decreases the effective transmit power of secondary transmitters by increasing the effective interference channel gain.

V. Conclusion

We analyzed the capacity of an opportunistic scheduling system in a spectrum sharing environment where there exist multiple secondary transmitters so that a secondary receiver can benefit from multi-user diversity effect. We considered three different transmit power control schemes of secondary transmitters according to power control capability and peak power constraints. When the transmit power of secondary transmitters is low compared to a given interference temperature, the adaptive transmit power scheme achieves almost the same capacity as that of the fixed transmit power scheme. Thus, it is not necessary to adaptively control secondary transmitters' transmit power in the low transmit power region. On the other hand, the adaptive transmit power scheme approaches the infinite transmit power scheme in terms of achievable capacity when the transmit power of secondary transmitters is high compared to a given interference temperature because secondary transmitters can not fully exploit their high peak power in the high transmit power region, where the effect of an interference temperature becomes dominant.

Appendix

Let $Z = \frac{Y}{X}$, $X = \alpha_i$, and $Y = Q\beta_i$, then $\Pr[Z \le z | X > A]$ can be derived as

$$\begin{aligned}
&\Pr[Z \le z | X > A] = \Pr[Y \le zX | X > A] \\
=&\int_A^\infty \int_0^{xz} \frac{1}{Q} e^{-\frac{y}{Q}} N_p e^{-x} \left(1 - e^{-x}\right)^{N_p - 1} dy dx \\
=&N_p \int_A^\infty e^{-x} \left(1 - e^{-x}\right)^{N_p - 1} \int_0^{xz} \frac{1}{Q} e^{-\frac{y}{Q}} dy dx \\
=&N_p \int_A^\infty \left(1 - e^{-x}\right)^{N_p - 1} \left(e^{-x} - e^{-\frac{Q+z}{Q}x}\right) dx \\
=&N_p \int_A^\infty \sum_{j=1}^{N_p} \binom{N_p - 1}{j - 1} (-1)^{j-1} \left(e^{-jx} - e^{-\frac{jQ+z}{Q}x}\right) dx \\
=&N_p \sum_{j=1}^{N_p} \binom{N_p - 1}{j - 1} (-1)^{j-1} e^{-jA} \left[\frac{1}{j} - \frac{Q}{Qj + z} e^{-\frac{A}{Q}z}\right].
\end{aligned}$$

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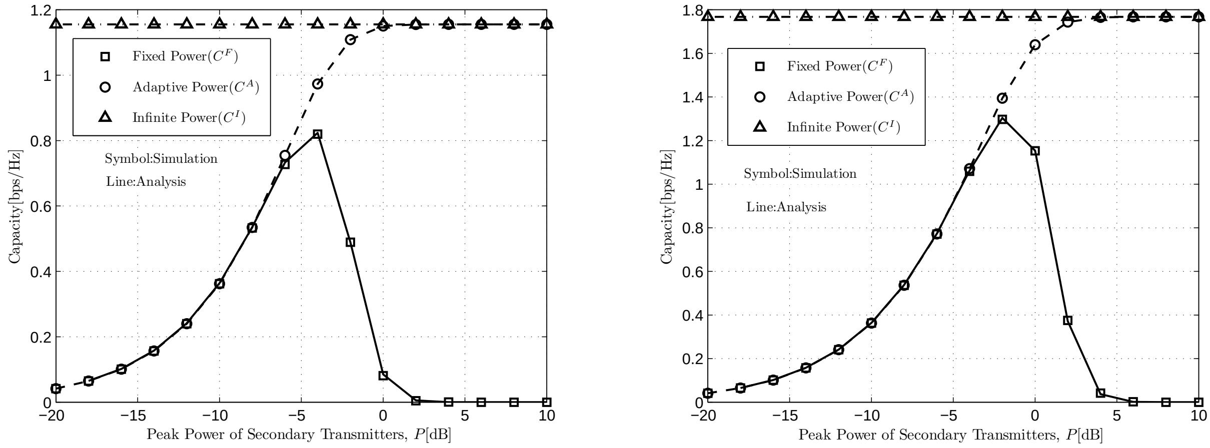
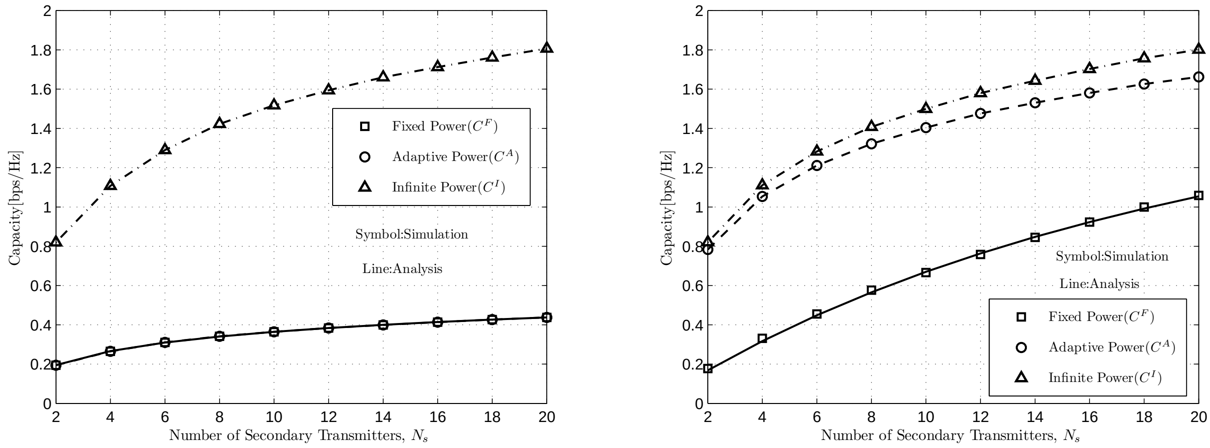


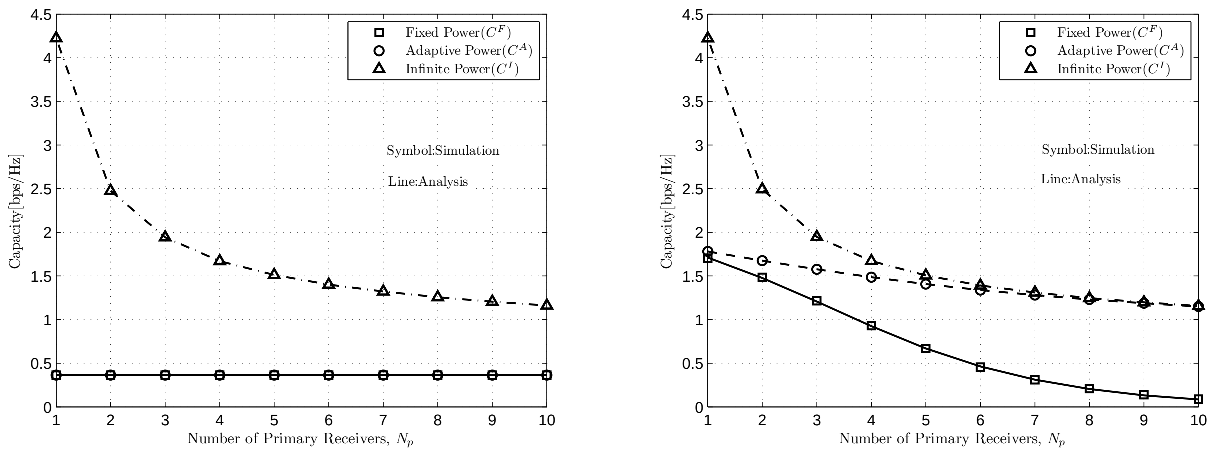
(a) $Q = 0$dB (b) $Q = 3$dB

Fig. 2. Average achievable capacity vs. P. $Q = 0$dB and $N_s = N_p = 10$.



(a) $P = -10$dB (b) $P = 0$dB

Fig. 3. Average achievable capacity vs. N_s. $Q = 0$dB and $N_p = 5$.



(a) $P = -10$dB (b) $P = 0$dB

Fig. 4. Average achievable capacity vs. N_p. $Q = 0$dB and $N_s = 10$.